

Technical Notes

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Rocket Propulsion Strategy Based on Kinetic Energy Management

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Introduction

THIS Note is a somewhat blue-sky tutorial on how one might increase the velocity increment (no-drag, constant-altitude scenario) of a rocket motor of a given mass fraction by replacing part of the propellant weight with an equal weight of low-molecular-weight working fluid (hydrogen), utilizing idealized heat exchange processes between the packet of hot propellant products and several packets of liquid hydrogen (LH_2), and optimizing sequencing of expulsion of the packets. The main principle invoked involves better matching of the product exit velocity to the changing vehicle velocity, so as to maximize the fraction of thermal energy converted to kinetic energy of the vehicle, rather than of the expelled gases. The overall objective is to convert more thermal energy to vehicle kinetic energy, as opposed to throwaway kinetic-plus-thermal energy of the exit gases. That is, it is desired to convert as much thermal energy to kinetic energy as possible, while simultaneously minimizing throwaway kinetic energy, by leaving the exhaust gases nearly motionless relative to the surroundings, their exit velocity relative to the vehicle being equal in magnitude to the vehicle velocity. As will be discussed, this approach does result in alteration of the fraction of thermal energy converted to kinetic energy, but this effect is far outweighed by the velocity matching effect, with a decrease in overall thermal energy conversion in some cases being accompanied by an increase in the velocity increment.

Analytical Approach

As indicated in the preceding text, this is a thermodynamic, very idealized exercise, and it is fully recognized by the author that the calculated performance gains will be impossible to realize in the real world, for reasons, such as an overwhelming increase in the required weight of the inerts to accomplish the individual steps, that will become obvious as the cycle is described. A brief description of the analysis approach follows. Note that all results involve zero initial missile velocity, though similar calculations can be easily carried out for other initial missile velocities.

In a baseline scenario, 100 lbm of propellant is burned at 1000 psia, and exhausted at matched conditions to an ambient pressure of 5 psia. The propellant is specified to have a flame temperature of 5000 K (no dissociation is considered because all calculations assume constant frozen properties), a product molecular weight of 30, and a specific heat ratio of 1.15. Stan-

dard frozen expansion calculations as presented in many textbooks¹ are applied for the calculation of specific impulse, exit temperature, and exit velocity relative to the vehicle, with a vehicle velocity increment then being calculated from $\Delta V = I_{sp} g_c \ln(M_{\text{initial}}/M_{\text{final}})$, an inert weight of 10 lbm being used for all scenarios. A differential form of this equation is used to calculate the vehicle velocity vs the cumulative amount of propellant expelled, with the gas exit velocity relative to the vehicle then being used to calculate the gas velocity relative to the surroundings and the exit gas kinetic energy vs the cumulative expelled weight; graphical integration is next used to calculate the total amount of kinetic energy of the exhaust gases. The sum of this quantity and the final vehicle kinetic energy is finally compared with the change in enthalpy of the gases from the chamber to exit to ensure energy conservation. Results are presented in line 7 of Table 1; the most important quantities are the velocity increment ΔV , 25,603 ft/s; the total thermal to kinetic energy conversion, 227,860 Btu; and the fraction of kinetic energy appearing as missile kinetic energy $|\Delta KE_{\text{missile}}/\Delta KE_{\text{total}}|$, 0.575.

In the second scenario examined, hot propellant is mixed with LH_2 (total weight of 100 lbm), with the mixture being expelled using the same chamber, exit, and ambient pressure as in the first scenario. The only additional calculations required for this case are straightforward frozen mixing calculations (including a term for the LH_2 heat of vaporization) of the resultant mixture properties (T , MW , c_p , γ), with specific impulse, velocity increment, etc., then being calculated as before using these values. Results are presented for seven ratios of propellant to LH_2 , including an all-propellant case, in lines 1–5 of Table 1. The fraction of kinetic energy appearing as missile kinetic energy is unchanged, whereas the thermal energy to kinetic energy conversion and the vehicle velocity increment first rise as hydrogen is substituted for propellant, and then fall with further increases in the hydrogen fraction.

In the third scenario examined, the one of interest, the propellant is first burned, with hot exhaust products being retained (impractical in the real world), and heat exchanged (not mixed) to thermal equilibrium with successive packets of LH_2 , these packets also being retained. Finally, the thermally depleted hot propellant products are expelled, followed by the coolest of the hydrogen packets, the second coolest hydrogen packet, etc., resulting in step increases in exit velocity relative to the vehicle with the release of each successive packet. One can apply either of two methods to analyze these processes: 1) a constant-volume approach or 2) a constant-pressure approach. The former approach is simpler mechanically but more complex mathematically, because blowdown analyses must be applied to the expansion processes. Because the overall exercise is already highly impractical, we add one more level of impracticality by use of a constant-pressure analysis. In this analysis, the solid propellant burns from nearly zero initial volume to some large final volume at 1000 psia, with the expansion work done being stored in some type of reservoir or spring. Then, during the heat exchange processes, energy is released from this reservoir as the product gases contract at constant pressure, and is stored in an additional reservoir associated with each hydrogen packet. Finally, during the exhausting of each packet of gas, the energy stored in each of the reservoirs is released to allow these processes to occur at constant chamber pressure.

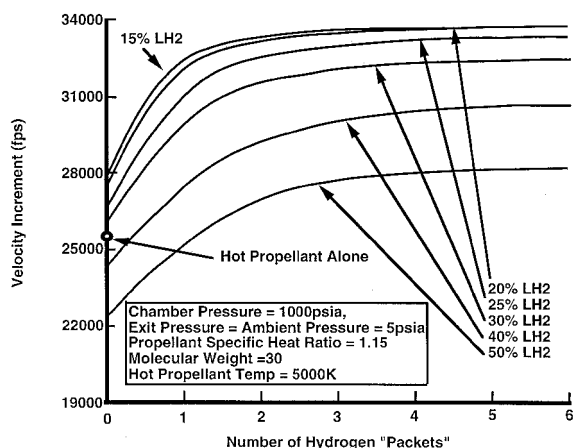
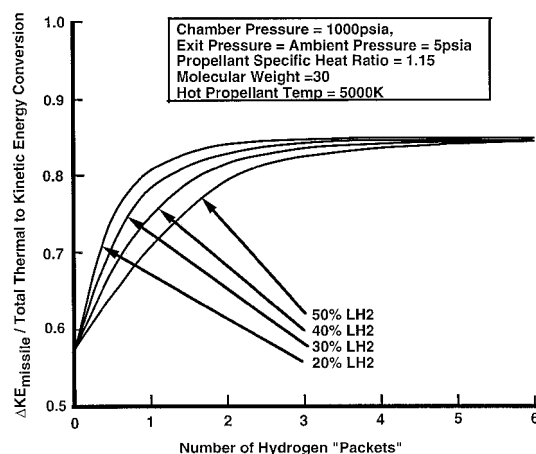
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Table 1 Summary of calculations: 10-lb inerts plus 100-lb total propellant (hot propellant + LH₂)^a

Description	Propellant weight, lbm	Hydrogen weight, lbm	Velocity increment, ft/s	Total impulse, lbf-s	Total kinetic energy, Btu	$ \Delta KE/\Delta \text{thermal energy} $	$\Delta KE_{\text{missile}}/\Delta KE_{\text{total}}$
Mixing cases, including all hot-propellant case	50	50	22,679	29,409	178,780	0.999	0.575
	60	40	24,499	31,769	208,640	0.999	0.575
	70	30	26,048	33,777	235,840	0.999	0.575
	80	20	27,232	35,313	257,770	0.999	0.575
	90	10	27,702	35,923	266,971	0.999	0.575
	95	5	27,286	35,383	259,005	0.999	0.575
	100	0	25,603	33,200	227,860	0.999	0.575
Heat exchange cases	50	1 × 50	25,142	25,808	176,520	0.998	0.716
	50	2 × 25	27,301	23,445	181,350	0.998	0.805
	50	5 × 10	27,984	20,715	183,840	0.999	0.851
	50	10 × 5	28,057	19,398	184,380	0.999	0.853
	60	1 × 40	27,757	27,201	204,850	0.998	0.752
	60	2 × 20	29,536	25,091	211,500	0.999	0.824
	60	4 × 10	30,296	23,196	215,030	0.999	0.853
	60	8 × 5	30,467	21,834	216,630	0.999	0.856
	70	1 × 30	30,101	28,360	229,940	0.998	0.788
	70	2 × 15	31,642	26,642	238,020	0.999	0.841
	70	5 × 6	32,390	24,805	243,860	0.999	0.860
	70	10 × 3	32,503	23,926	245,920	0.999	0.859
	75	1 × 25	31,095	28,897	240,550	0.998	0.803
	75	2 × 12.5	32,458	27,430	248,890	0.999	0.846
	75	5 × 5	33,136	25,932	255,196	0.999	0.860
	75	10 × 2.5	33,268	25,238	257,550	0.999	0.859
	80	1 × 20	31,874	29,439	249,150	0.999	0.815
	80	2 × 10	33,015	28,263	257,230	0.999	0.847
	80	4 × 5	33,519	27,353	262,330	0.999	0.856
	80	8 × 2.5	33,712	26,760	265,250	0.999	0.856
	85	1 × 15	32,286	30,032	255,115	0.999	0.818
	85	2 × 7.5	33,158	29,181	262,055	0.999	0.839
	85	3 × 5	33,428	28,791	264,900	0.999	0.844
	85	6 × 2.5	33,671	28,320	268,109	0.999	0.846

^aChamber pressure = 1000 psia, exit pressure = ambient pressure = 5 psia, propellant flame temperature = 5000 K, specific heat ratio = 1.15, MW = 30.

**Fig. 1 Velocity increments for various scenarios.****Fig. 2 Fraction of kinetic energy to the missile.**

Enthalpy balance calculations are used to calculate the temperature of each packet of hydrogen and of the thermally depleted hot propellant gas packet. As before, the molecular weight and specific heat ratio of the hot propellant product gases are set at 30 and 1.15, and these values for the hydrogen packets are set at 2 and 1.4. The same expansion calculations used in the earlier scenarios are then used to calculate specific impulse, exit gas velocity relative to the vehicle, and exit temperature for gases from each packet. The depleted hot-propellant products are expelled first; a velocity increment for this step is calculated using the specific impulse for this packet and a mass ratio equal to the total system initial weight, 110 lbm, divided by the weight remaining after expulsion of this gas. Next, the coolest hydrogen packet is expelled, with the second velocity increment being calculated using its specific impulse and a mass ratio equal to system weight after expulsion of the

hot-propellant products, divided by the weight subsequent to expulsion of this second packet. This process is repeated for all packets, with the total velocity increment being the sum of the individual increments. During each step, vehicle velocity vs cumulative mass expelled is tracked and used with the exit gas velocity relative to the vehicle during that step, to calculate a history of exit gas velocity relative to the surroundings vs mass expelled (which will include step changes as each new packet is tapped). These velocities are then used to calculate an exit gas kinetic energy per unit mass vs cumulative mass expelled history, which is integrated to yield the total amount of exhaust gas kinetic energy for a given case. As before, this amount is added to the final vehicle kinetic energy for comparison with the total enthalpy loss associated with all packets. Results of a number of cases, involving various ratios of hot-propellant mass to LH₂ mass and various numbers of hydrogen

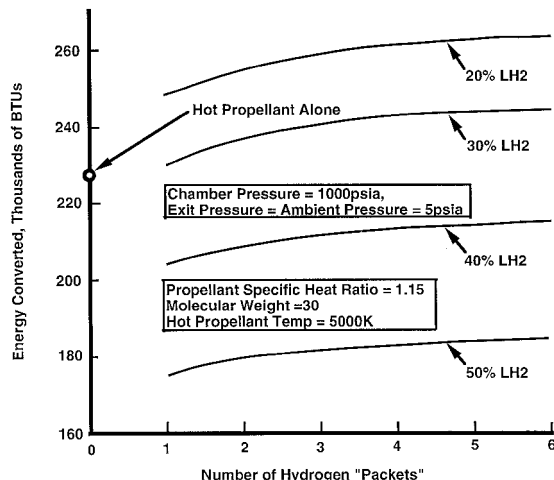


Fig. 3 Conversion of thermal to kinetic energy.

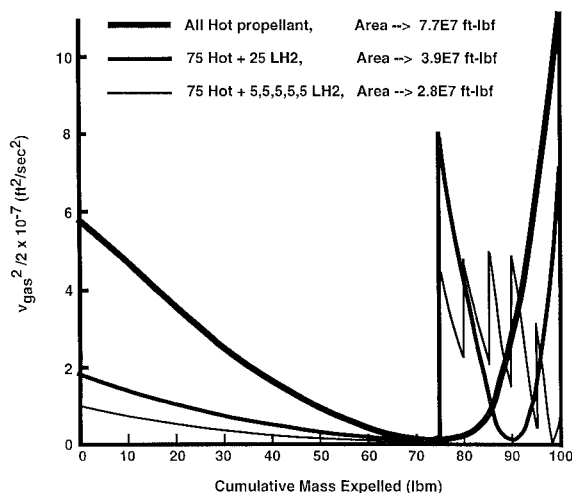


Fig. 4 Kinetic energy profile of exhaust gases.

packets (all of equal mass for a given case), are presented in Table 1 and Figs. 1-4.

Results and Discussion

As may be seen from Table 1 and Fig. 1, considerable increases in velocity increment relative to the baseline case can be achieved with the heat-exchange scenario, the level of increase depending on both the hot-propellant/LH₂ ratio and the number of sequential LH₂ packets utilized, though there is relatively little increase in velocity increment beyond three packets. The maximum increase in ΔV relative to the baseline value is approximately 32%. Careful examination of Table 1 and Figs. 1-3 reveals changes in both thermal to kinetic energy transformation and the fraction of kinetic energy going to the vehicle. At low hot-propellant/LH₂ ratios, there is a significant decrease in the thermal energy-kinetic energy conversion relative to the baseline case, this conversion increasing monotonically with increasing value of this ratio up to about a 90/10 ratio. At this ratio, the conversion is approximately 17% higher than the baseline case, with little dependence on the number

of hydrogen packets. The fraction of kinetic energy delivered to the vehicle (Fig. 2 and the last column of Table 1) depends significantly on both the hot-propellant/LH₂ ratio and the number of hydrogen packets (up to three). As pointed out earlier, there are a number of cases for which, even with a decrease in the thermal energy-kinetic energy conversion relative to the baseline, the velocity increment is larger than the baseline value. (Note particularly the case of the 60/40 ratio with four hydrogen packets.)

How the kinetic energy being wasted in the exhaust gases changes from the all-hot-propellant case to the 75/25 hot-propellant/LH₂ case, for one hydrogen packet and five hydrogen packets, is demonstrated in Fig. 4, where kinetic energy per unit mass is plotted against cumulative mass expelled for these three cases. (Recall that, because initial vehicle velocity is zero in these calculations, initial kinetic energy associated with material later expelled is also zero.) The heavy curve, representing the baseline hot-propellant case, decreases monotonically from zero mass expelled to approximately 70-75% mass expelled, where exit gas velocity relative to the vehicle (constant) and vehicle velocity (monotonically increasing) become equal, resulting in zero gas velocity relative to the surroundings. Beyond that point the curve turns up, as exhaust gas velocity relative to the vehicle falls below vehicle velocity, resulting in increasing exhaust gas velocity relative to the surroundings. (Both leading and lagging velocities result in positive kinetic energy values.) The area under the curve yields the total kinetic energy of the exhaust gases, in this case approximately 7.7×10^7 ft-lbf. The one-hydrogen-packet case is represented by the intermediate weight curve; as may be seen, kinetic energy per unit mass starts at a considerably lower value because of the lower exit velocity of the thermally depleted propellant products, decreases again to zero as vehicle velocity catches up with exit velocity, takes a sudden steep jump as the higher exit velocity hydrogen exhaust stream appears, decreases again as vehicle velocity again catches up with exit velocity, and finally again increases until burnout. Integration of the area under this curve yields a value for the total kinetic energy of the exhaust gases of approximately 3.9×10^7 ft-lbf, just over half the baseline case value. Finally, the lightest curve represents the case of 75 lbm of hot propellant plus five 5-lbm packets of hydrogen; in this case, there are five discontinuous energy jumps as the successively hotter hydrogen exhaust streams appear. Integration of the area under this curve yields an exhaust gas total kinetic energy value of approximately 2.8×10^7 ft-lbf, a further decrease of 30% from the intermediate case. These three cases appear as lines 7, 20, and 22 in Table 1; as may be seen, the decreases in kinetic energy of the exhaust gases result in the fraction of total kinetic energy associated with the vehicle (last column of Table 1) changing from 0.575 to 0.803 to 0.860.

As stated earlier, the author does not mean to imply any practical applicability of this approach to increasing the velocity increments of rocket vehicles, but is merely presenting what he regards as an interesting tutorial on the thermodynamics of the situation. Further studies examining a constant-volume analysis or making some attempt to factor in effects of increased inert weights associated with implementation of such an approach are left as an exercise for the reader.

Reference

¹Sutton, G. P., *Rocket Propulsion Elements*, 6th ed., Wiley, New York, 1992.